<u>Example Projectile Motion Problems</u>
Physics Phun!

a) A mortar shell projected at an angle of 65° above the horizontal strikes a tower 25 m away at a point 15 m above the point of projection. Find the initial speed v_0 of the projectile.	$\Delta s_x = v_{0x}t = v_0\cos\theta_0 t \quad t = \frac{\Delta s_x}{v_0\cos\theta} = \frac{25m}{v_0\cos 65}$ $\Delta s_y = v_0\sin\theta_0 t + \frac{1}{2}gt^2 \quad 15m = v_0\sin 65\left(\frac{25m}{v_0\cos 65}\right) + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)\left(\frac{25m}{v_0\cos 65}\right)^2$ $\boxed{v_0 = 21.1\frac{m}{s}}$
b) How long was it in the air?	$t = \frac{\Delta s_x}{v_0\cos\theta} = \frac{25m}{\left(21.1\frac{m}{s}\right)\cos 65} = \boxed{2.8s}$
c) Find the magnitude and direction of the velocity of the projectile when it strikes the tower.	$v_x = v_0\cos\theta = \left(21.1\frac{m}{s}\right)\cos 65 = \boxed{8.92\frac{m}{s}}$ $v_y = v_0\sin\theta + a_y t = \left(21.1\frac{m}{s}\right)(\sin 65) + (-9.8\frac{m}{s^2})(2.8s) = \boxed{-8.32\frac{m}{s}}$ $v = \sqrt{\left(8.92\frac{m}{s}\right)^2 + \left(-8.32\frac{m}{s}\right)^2} = \boxed{12.2\frac{m}{s}} \quad \tan\theta = \frac{-8.32\frac{m}{s}}{8.92\frac{m}{s}} \quad \boxed{\theta = 43.1^o \text{ below pos.x}}$
d) Find the maximum height of the projectile.	$v^2 = v_0^{\ 2} + 2a\Delta s \quad 0 = \left(\left(21.1\frac{m}{s}\right)(\sin 65)\right)^2 + 2\left(-9.8\frac{m}{s^2}\right)h \quad \boxed{h = 18.7m}$
e) If the projectile had not struck the tower, what would have been the range R of the projectile?	$y = 0 \quad 0 = v_{0y}t + \frac{1}{2}at^2 = \left(21.1\frac{m}{s}\right)(\sin 65)t + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)t^2$ $t = \frac{\left(21.1\frac{m}{s}\right)(\sin 65)}{\frac{1}{2}\left(9.8\frac{m}{s^2}\right)} = 3.9s \quad R = v_0 t = \left(21.1\frac{m}{s}\right)(\cos 65)(3.9s) = \boxed{34.8m}$
f) What would be the magnitude and direction of the velocity of the projectile upon impact?	$\boxed{v_x = 8.92\frac{m}{s}}\text{(constant)} \quad v_y = v_0\sin\theta + a_y t = \left(21.1\frac{m}{s}\right)(\sin 65) + \left(-9.8\frac{m}{s^2}\right)(3.9s) = \boxed{-19.1\frac{m}{s}}$ $v = \sqrt{\left(8.92\frac{m}{s}\right)^2 + \left(-19.1\frac{m}{s}\right)^2} = \boxed{21.1\frac{m}{s}} \quad \tan\theta = \frac{-19.1\frac{m}{s}}{8.92\frac{m}{s}} \quad \boxed{\theta = 65^o \text{ below pos. x}}$

An antiaircraft artillery gun fires a projectile with a muzzle velocity of 1000 m/s. If the projectile is to explode at an altitude of 4000 m and a horizontal range of 3000 m from the gun site, find the angle of elevation of the gun and the fuse setting of the projectile.	$v_0 = 1000\,\frac{m}{s} \quad \Delta y = 4000m \quad \Delta x = 3000m$ $a_x = 0 \quad a_y = -9.8\,\frac{m}{s^2}$
Find the firing angle.	$\Delta x = v_{0x}t \quad t = \frac{3000m}{\left(1000\,\frac{m}{s}\right)(\cos\theta)}$ $\Delta y = v_{0y}t + \frac{1}{2}at^2$ $4000m = \left(1000\,\frac{m}{s}\right)(\sin\theta)\left(\frac{3000m}{\left(1000\,\frac{m}{s}\right)(\cos\theta)}\right) + \frac{1}{2}\left(-9.8\,\frac{m}{s^2}\right)\left(\frac{3000m}{\left(1000\,\frac{m}{s}\right)(\cos\theta)}\right)^2$ $4000 = \frac{\sin\theta}{\cos\theta}(3000) - \frac{44.1}{\cos^2\theta}$ Multiply by $\cos^2\theta$ $4000\cos^2\theta + 44.1 = 3000\sin\theta\cos\theta$ Divide by 1000 and let $\sin\theta = \sqrt{1-\cos^2\theta}$ $4\cos^2\theta + 0.0441 = 3\cos\theta\sqrt{1-\cos^2\theta}$ Square both sides and collect similar terms $16\cos^4\theta + 0.353\cos^2\theta + 0.00195 = 9\cos^2\theta - 9\cos^4\theta$ $25\cos^4\theta - 8.65\cos^2\theta + 0.00195 = 0$ Use the quadratic equation $\cos^2\theta = \frac{8.65 \pm \sqrt{(8.65)^2 - 4(25)(0.00195)}}{2(25)} = 0, 0.346$ $\boxed{\theta = 54^o}$
Find the time until detonation.	$t = \frac{3000m}{\left(1000\,\frac{m}{s}\right)(\cos 54)} = \boxed{5.1s}$