Examining Change

Unit Summary

How do things grow? How do they change? How can we use mathematics to predict change? What does it mean to grow linearly? In this unit, students explore various growth patterns and make generalizations by reasoning about the structure of patterns. Students use their generalizations to make mathematical conjectures backed by data and reasoning, and they use the *Showing Evidence Tool* to organize their conjectures. Students conclude the unit by presenting their conjecture, data, and reasoning using a multimedia presentation. Students then self-evaluate their conjectures and presentations for strengths and areas in which they can improve as well as decide which conjecture best represents the problem context.

At a Glance
Grade Level: 6-10
Subject: Mathematics
Topics: Algebra
Higher-Order Thinking Skills: Reasoning, Evaluation, Generalizing
Key Learnings: Conceptualization and Representation of Linear Functions, Making Conjectures, Using Inductive and Deductive Reasoning, Developing and Evaluating Mathematical Arguments
Time Needed: 8-10 hours
Background: California, United States

Curriculum-Framing Questions

- **Essential Question**
  What happens next?
- **Unit Questions**
  How do we predict growth?
- **Content Questions**
  What is linear growth?
  How do you represent and analyze mathematical situations and structures using algebraic symbols?
  How do you use mathematical models to represent and understand quantitative relationships?
  How does change in one variable relate to change in a second variable?

Things You Need
Assessment
Standards
Resources

Assessment Processes

View how a variety of student-centered assessments are used in the Examining Change Unit Plan. These assessments help students and teachers set goals; monitor student progress; provide feedback; assess thinking, processes, performances, and products; and reflect on learning throughout the learning cycle.

Instructional Procedures

Introducing the Unit

This unit can be used to introduce students to linear functions. The problems students investigate build on their prior work with patterns and use geometric contexts that allow visual and conceptual access to the concepts of slope and y-intercept.

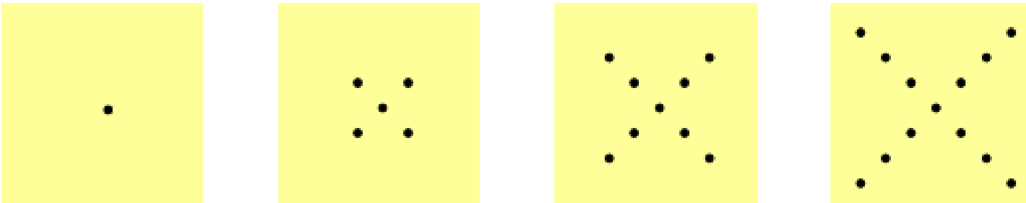
Begin the unit by writing on the board the Essential Question, *What happens next?* Ask students to reflect on the question and how it relates to mathematics and patterns. Guide students to think about how they use patterns to help them predict what happens, and ask students to give examples of how they have used patterns to describe, extend, and make generalizations.

Pose the Unit Question, *How do we predict growth?* Have students propose some ways that things grow. Provide examples of growth patterns. Encourage students to share their thoughts as you record their responses on the board. Try to elicit from students a variety of patterns—pointing out to students the difference between a growing pattern (such as, growing by adding 4 each time) and a repeating pattern (such as, even, odd, even, odd)

Analyzing Change in Various Contexts

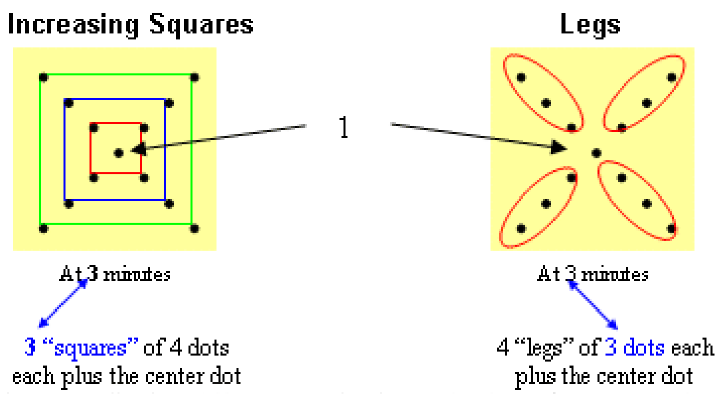
Growing By Dots Lesson

- Begin the lesson by posting a sheet of paper with 1 black dot, then a sheet with 5 dots, then one with 9 dots, then one with 13 dots on the blackboard or whiteboard (using tape or magnetic strips).



At the Beginning | At 1 minute | At 2 minutes | At 3 minutes

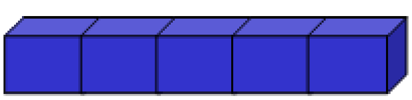
- Ask students: *What is changing, growing?* Elicit several student responses (number of dots increases four at a time, squares are increasing, dots are increasing away from the one in the center, and so forth). Ask students to think of increasing patterns in their everyday lives. Then have students think of a specific context in everyday life that the growing dot pattern might represent (such as saving for a purchase, traffic buildup at rush hour, virus increase, and so forth).
- After deciding on a context for the growing dots, ask students to come up with a problem to solve—such as, at 48 minutes, how many viruses are there? Have students work on the problem individually, and then have students discuss the problem with partners or in small groups. Walk around during this time asking probing questions and monitoring the methods students are using to find their solutions, noting if students are approaching the problem using the following two common but distinct methods:



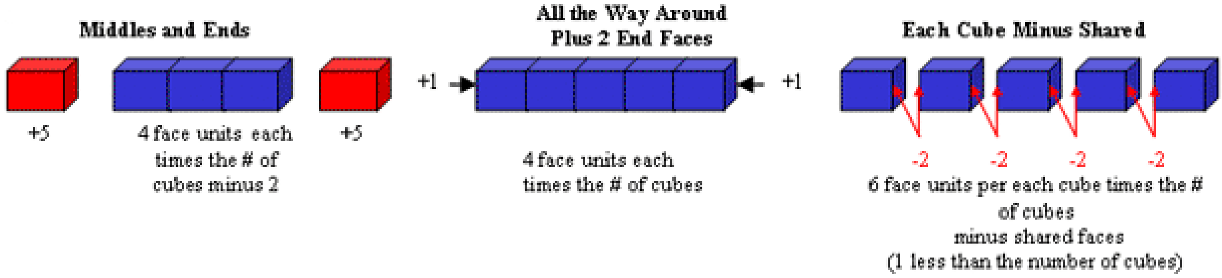
- Choose students to display different methods on the board, encouraging the use of color to distinguish parts of the method. Ask the class how many other students used the same method. Then, ask if each method always works and why. Have students compare and contrast the methods. At this point, students do not need to come up with formalized algebraic rules to represent their generalizations. The important point is to focus on how written and verbal rules correspond to the geometric model of the dots—what is increasing each minute (a square of four dots or a dot in each of the four arms). Understanding how the dots are increasing serves as the basis for conceptualizing the rate of change in the linear function.

Cubes Train Lesson

- Materials needed: cubes for each group
- Begin the lesson by posing the following problem:
  - *How many face units are there when you put 100 cubes together in a train (sharing a face)? How many face units are there when you put any number of cubes together?*



  - Have students work on the problem individually, and then have students discuss the problem with partners or in small groups. Walk around during this time to see what solutions students are using, noting if students are approaching the problem in different ways. Use the observation checklist to note students' mathematical thinking, processes and behaviors. Decide which students will share their solution methods or reasoning during the whole class discussion (and in what order). Some typical ways that students approach this task are:



- Choose students to display different methods on the board, encouraging the use of color to distinguish parts of their method. Ask the class how many other students used the same method. Then, ask the class if each method always works, and why. Ask the class to compare and contrast the methods. Push for students to translate their words and numeric expressions into more formal algebraic notation, such as, 4(n-2) + 5 + 5, 4n + 2, and 6n- 2(n-1).
- For homework (and assessment of student learning), pose the following problem to students:

Compare and contrast the dots and cubes patterns—what was the same, what was different? How did you predict growth of these patterns? Create a dots pattern to represent the expression: 4n + 2.

Review homework the next day and modify instruction as necessary.

Practice Using the Tool
Introduce students to the *Showing Evidence Tool* by exploring the Try the Tool demonstration space together. Discuss the sample argument together, or create a sample project and show students how to add, describe, and rate evidence and claims. Model how student teams peer review each other's work. Also, show the Comments feature, and agree on how it will be used. Note that the issue of rating sources of evidence, while important in other explorations, is not as important for this unit's mathematical explorations, because the source is the problem being investigated. In contrast, the mathematical argument students use to back up their conjectures is important, because the conjectures must be based on evidence or data.

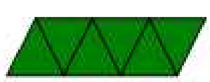
Hold a discussion around the idea of a mathematical argument using data as evidence. Ask students to consider the following questions as they work on their mathematical arguments:

- *Does my data back up my conjecture or generalization?*
- *Is my data complete enough to follow a logical mathematical argument?*
- *Can someone make a counterargument that will weaken my conjecture?*
- *Will my evidence convince a skeptic?*

Distribute the conjecture and evidence checklist to help guide students' initial work with the tool.

Set Up the Project
Before proceeding with the next activity, click here to set up the Polygon Problem project in your workspace. Provide polygons (using pattern blocks or paper shapes, such as triangles, squares, pentagons, and hexagons) for students to use during the investigation. Then, pose the following problem to students:

Polygon Investigation



If you line up 100 equilateral triangles in a row, what will the perimeter be? Find a rule for any number of triangles.

What if you lined up:
- Squares?
- Regular pentagons?
- Regular hexagons?
- n-sided polygons?

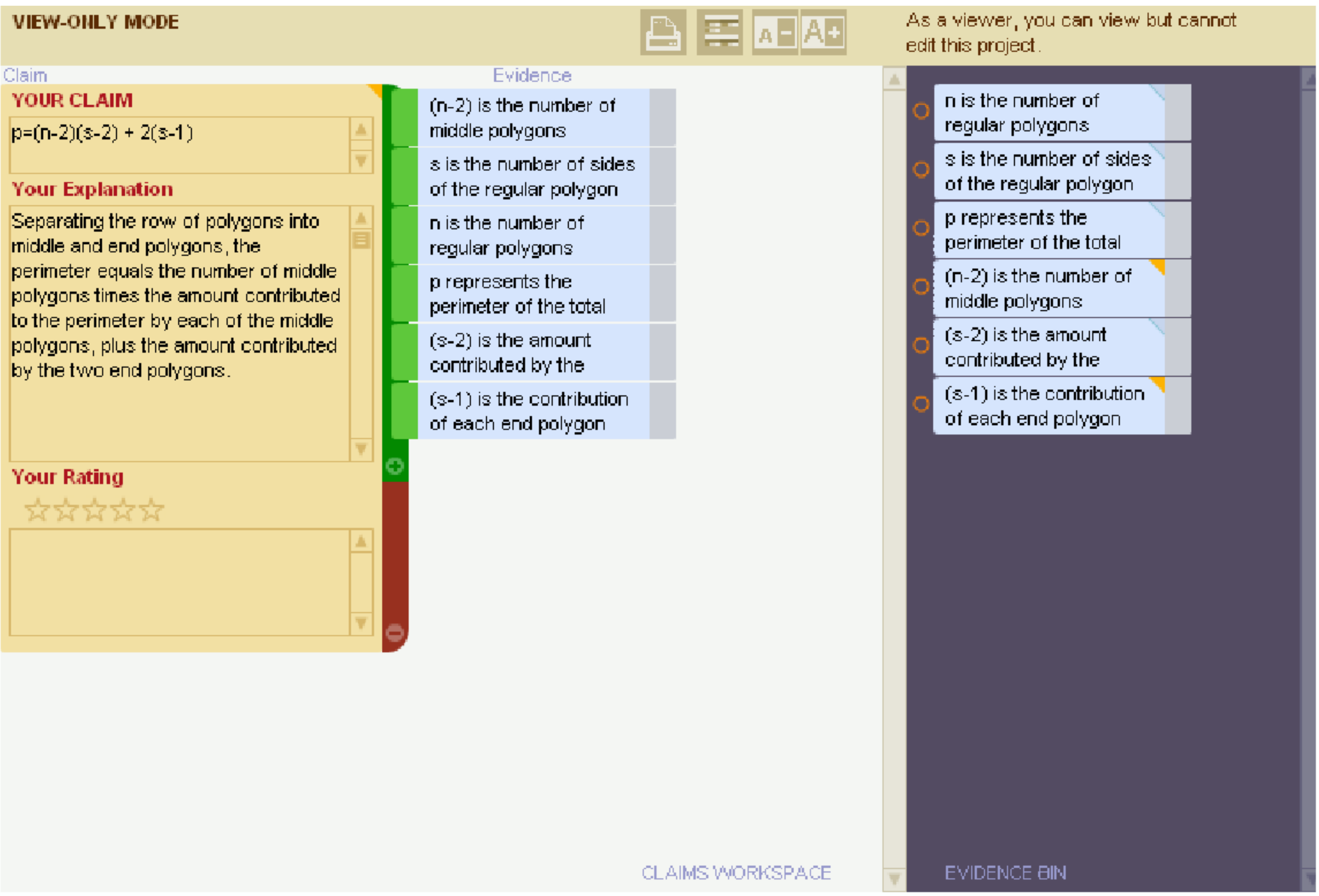
Use the Tool
Have students log into their *Showing Evidence* team space. Assign students to work with a partner. Instruct students to explore the problem, beginning with the triangles and investigating all of the polygons in order to find a rule for any polygon. Have students create their conjectures in the *Showing Evidence* workspace. Give parameters regarding the amount of data needed for each conjecture. For example, explain that students must find at least four pieces of evidence or data that supports the conjecture, and the evidence must form a logical mathematical argument. Remind students that evidence must back up their work on the problem and must not be based on the students' opinions or guesses.

After students complete the initial problem-investigation stage, use the teacher workspace in *Showing Evidence* to assign each group of partners a peer group to review. Tell peer review groups to read and assess the conjectures of the group assigned to them, and make constructive comments and corrections where needed to the conjectures and evidence. Students can comment on the work by requesting clarification of evidence, pointing out where conjectures are unclear, identifying questionable facts or assumptions, and correcting distortions of opposing points of view. Reviewers can use the conjecture and evidence checklist to guide this work.

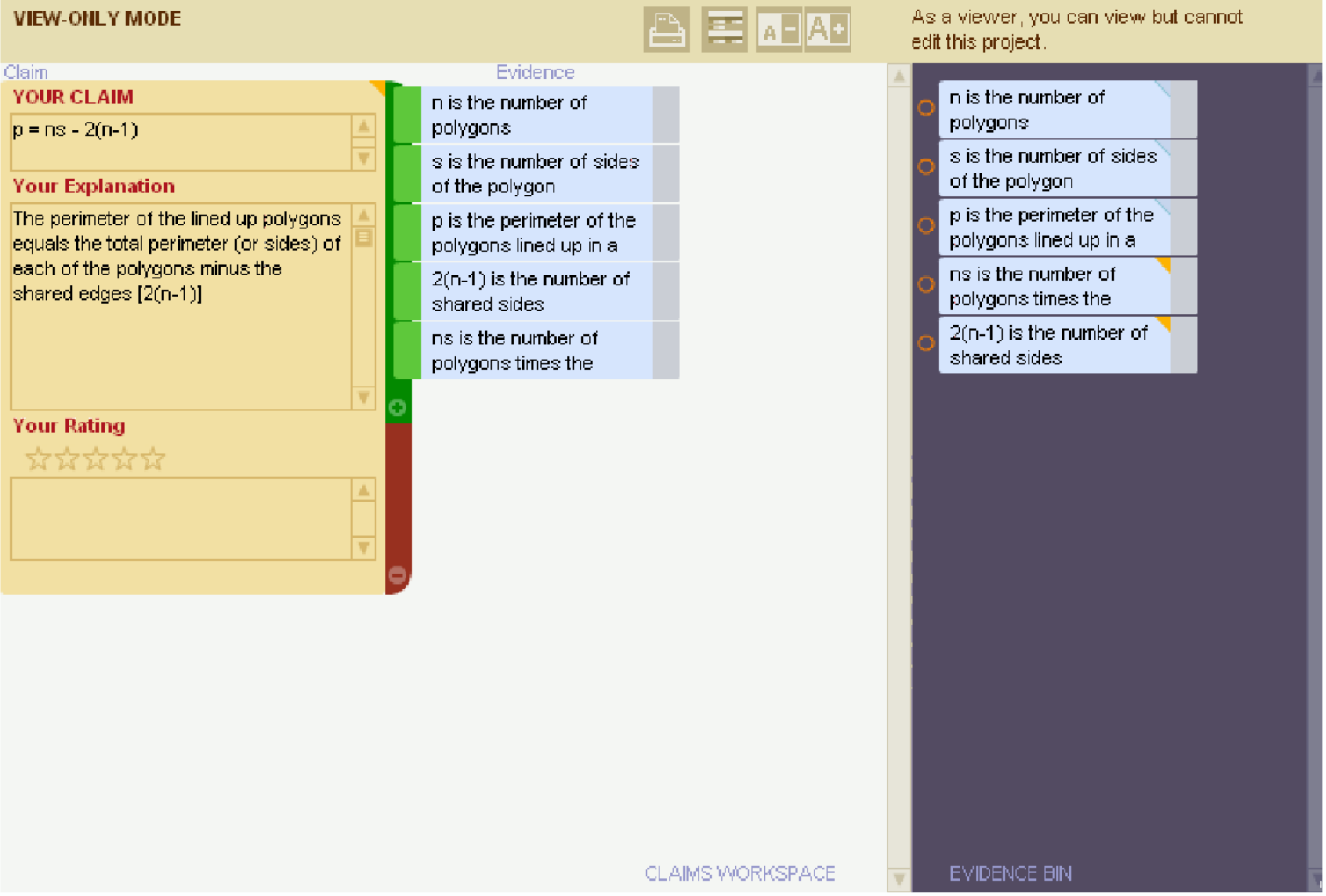
Use the Comments feature and notes from the observation checklist to provide feedback, redirect effort, suggest new avenues of study, or ask for clarification about a team's thinking. When the review process is complete, give the students time to make adjustments and corrections based on the comments from the peer review.

Examine the Showing Evidence Activity
The *Showing Evidence Tool* space below represents two different teams' investigations in this project. The cases you see are functional. You can double-click on the evidence and comments to read the teams' descriptions.

Project Name: Polygon Problem (Click here to set up this project in your workspace)
Prompt: What is the perimeter of any number of n-sided regular polygons lined up in a row?



Project Name: Polygon Problem (Click here to set up this project in your workspace)
Prompt: What is the perimeter of any number of n-sided regular polygons lined up in a row?



Revisiting Student Conjectures
Ask each group to give a multimedia presentation detailing their conjectures using the project rubric as a guide. Have students take notes during the presentations. After the presentations, have the class work together to create a list on the board of the different conjectures made by the student groups (students may have overlapping methods). Students can then self-assess their own conjectures and presentations for strengths and areas needing improvement. During their assessments, have students decide which conjecture best represents the problem context.

Reflecting on the Unit
Again, ask students the Essential Question: *What happens next?* In small groups have students discuss the question in relation to what they have learned with their investigations and discussions. Bring the discussion back to the whole group and give students an opportunity to share what they talked about.

Prerequisite Skills

- Experience in analyzing patterns
- Experience in generalizing from patterns
- Experience in using various mathematical representations

Differentiated Instruction

Resource Student

- Provide extra time for investigating problems
- Reduce the amount of evidence required
- Provide adaptive technologies
- Make available support from resource specialists
- Modify learning objectives to reduce the level of depth and complexity required in final products

Gifted Student

- Give the student the freedom to explore mathematical offshoots of the investigations that intrigue the student and to craft evidence in new and original ways.
- During the discussion on the conjectures, encourage the student to explore in detail the similarities and differences among the conjectures and to craft extensive arguments as to which of the methods best represents the context of the polygon problem.

English Language Learner

- Group the student with bilingual students and work with English language instruction specialists.

- Use multiple representations (especially visual and tactile) to help with mathematical access to problems and methods.

Credits

A teacher contributed this idea for a classroom project. A team of educators expanded the plan into the example you see here.

Assessment Plan

Assessment Timeline

Before project work begins

Students work on projects and complete tasks

After project work is completed

- Questioning

- Anecdotal Notes
- Review
- Homework

- Observation Checklist
- Conjecture and Evidence Checklist
- Questioning

- Peer Review
- Comments in the *Showing Evidence Tool*
- Project Rubric

- Project Rubric

- Reflective Discussion

Assess the quality and complexity of students' conjectures, evidence, and construction of a mathematical argument using the work in the *Showing Evidence Tool.* Throughout the unit, conduct daily assessments of student collaboration, mathematical understanding, and collaboration skills using an observation checklist. Provide opportunities for students to peer review using the conjecture and evidence checklist as a guide. Use the project rubric to assess students' final products.

Content Standards and Objectives

Targeted Content Standards and Objectives

National Council of Teachers of Mathematics Standards

Algebra (Grades 6-8)

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Relate and compare different forms of representation for a relationship
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
- Develop an initial conceptual understanding of different uses of variables
- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
- Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use graphs to analyze the nature of changes in quantities in linear relationships

Reasoning and Proof (Grades 6-8)

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof
- Examine patterns and structures to detect regularities
- Formulate generalizations and conjectures about observed regularities
- Evaluate conjectures
- Construct and evaluate mathematical arguments

Student Objectives

Students will be able to:

- Relate and compare different forms of representation for a relationship
- Develop an initial conceptual understanding of slope and y-intercept
- Use symbolic algebra to represent situations and to solve problems, specifically those that involve linear relationships
- Formulate, investigate, and evaluate mathematical conjectures
- Select and use various types of reasoning and methods of proof
- Examine patterns and structures to detect regularities
- Formulate generalizations and conjectures about observed regularities
- Construct and evaluate mathematical arguments

Resources

Materials and Resources

Supplies

- Cubes
- Pattern blocks or tagboard polygon shapes
- Colored pencils for students; colored chalk or markers for presentations of solutions

Technology – Hardware

- Computer with Internet connection to access the *Showing Evidence Tool*
- Projection system to show students how to use the *Showing Evidence Tool*

Technology – Software

- Multimedia software to create presentation of conjectures

Math Observation Checklist

	Notes
COMMUNICATES MATHEMATICALLY	
☐ Shares ideas ☐ Asks appropriate questions ☐ Shows work ☐ Explains process ☐ Justifies answers	
THINKS, REASONS, PROBLEM SOLVES	
☐ Determines the problem ☐ Chooses appropriate strategies ☐ Applies strategies effectively ☐ Checks answer ☐ Generalizes process to other situations ☐ Sees relationships between concepts	
MATH BEHAVIORS	
☐ Plans and revises ☐ Demonstrates task commitment ☐ Uses more than one strategy to solve ☐ Uses mental math	

Conjecture and Evidence Checklist

____ Is our conjecture clear?

____ Does our data back up our conjecture or generalization?

____ Is our evidence based on data, not opinions or guesses?

____ Is our conjecture complete enough to follow a logical mathematical argument?

____ Can someone make a counter argument that will weaken our conjecture?

____ Will our evidence convince a skeptic?

____ Have we included the reasoning behind our conjecture?

Project Rubric

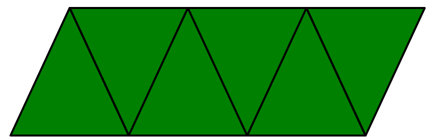
	4	3	2	1
Reasoning and Proof	• Conjecture follows a logical mathematical argument. • Deductive arguments are used to justify decisions. • Accurate, credible evidence is used to justify and support decisions made and conclusions reached. • Testing and accepting or rejecting of conjecture is shown with well-thought-out rationale for decision. • Generalization is made to other cases.	• Arguments are constructed with adequate mathematical basis. • A systematic approach and/or justification of correct reasoning is present. • Patterns are explained. • Testing and accepting or rejecting of conjecture is shown.	• Arguments are made with some mathematical basis. • Some correct reasoning or justification for reasoning is present through trial and error or by unsystematically trying several cases.	• Arguments are made with no mathematical basis. • Neither correct reasoning nor justification for reasoning is present.
Mathematical Communication	• A strong sense of audience and purpose is communicated. • Arguments are supported by mathematical properties used. • Precise math language and symbolic notation are used to communicate ideas.	• A sense of audience or purpose is communicated. • Communication of an approach is evident through a methodical, organized, coherent, sequenced, and labeled response. • Formal math language is used throughout the solution to share and clarify ideas.	• Some awareness of audience or purpose is communicated. • Some communication of an approach is evident through explanations. • Some formal math language is used, and examples are provided to communicate ideas.	• No awareness of audience or purpose is communicated. • Little or no communication of an approach is evident. • Everyday, familiar language is used to communicate ideas.
Presentation	• The presentation is highly effective. • The ideas are presented in an engaging, polished, clear, and thorough manner and are mindful of audience, context, and purpose. • The use of the presentation tool enhances the communication of the content.	• The presentation is effective. • The ideas are presented in a clear and thorough manner, showing awareness of the audience, context, and purpose. • The use of the presentation tool supports the communication of the content.	• The presentation is somewhat effective. • Some problems with clarity, thoroughness, delivery, and polish are evident. • The ideas are presented in a way that is unclear whether the audience, context, and purpose are considered. • The use of the presentation tool does not support the communication of the content.	• The presentation is ineffective. • The presentation is unpolished, providing little evidence of planning, practice, and consideration of purpose and audience. • The use of the presentation tool interferes with the communication of the content.

Some content adapted from: www.exemplars.com/resources/rubrics/nctm.html*.

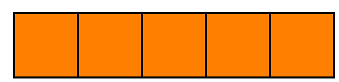
Polygon Investigation

By Jennifer and Brandon

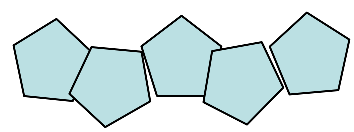
The Problem



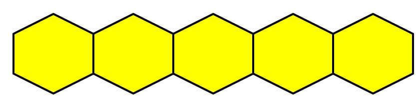
- If you line up 100 equilateral triangles in a row, what will the perimeter be? Find a rule for any number of triangles.
- What if you lined up:
  - Squares?



  - Regular Pentagons?

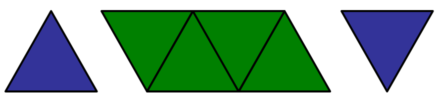
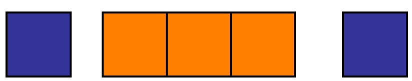
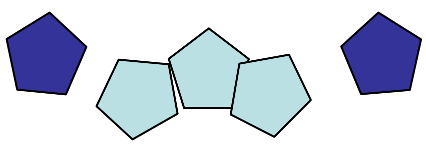
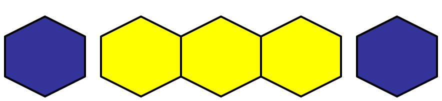


  - Regular Hexagons?



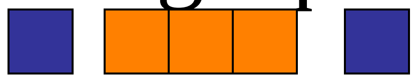
  - n-sided Polygons?

Our Mathematical Process

End + Middles + End	**# of Sides**	**Each End Contributes**	**Middles Contribute**
	3	3-1	3-2
	4	4-1	4-2
	5	5-1	5-2
	6	6-1	6-2
	n	n-1	n-2

Our Generalized Claim

- Visually: (using squares)



Each end contributes 3 sides

- Algebraically:

$$p = (n-2)(s-2) + 2(s-1)$$

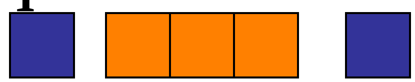
The number of middle squares (2 less than the total)

Each middle contributes 2 sides

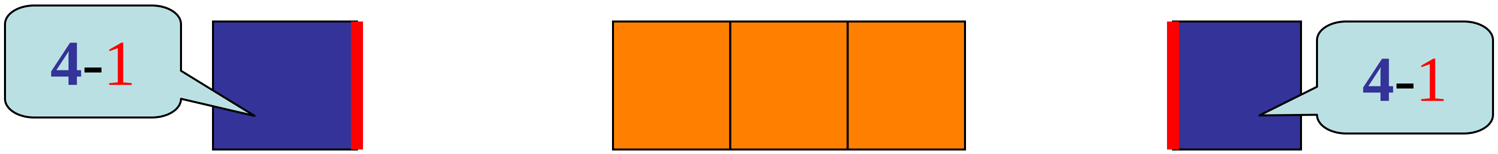
- Verbally:

  Separating the row of polygons into **middle** and **end** polygons, the perimeter equals the **number of middle polygons** times the **amount contributed to the perimeter by each of the middle polygons**, plus **the amount contributed by the two end polygons**.

Our Evidence

- Still using squares to visually support our evidence: 
- Our variables represent:
  - p= perimeter of total
  - s= # of sides (squares have 4)
  - n= the # of squares

Our Evidence



- **(n-2)** equals the number of middle polygons (in this case there are 5-2 middle squares)
- **(s-2)** equals the amount of sides each middle polygon contributes to the perimeter (in this case each square in the middle contributes 2--one top and one bottom side)
- **2(s-1)** equals the two end polygons--each contributing the # of sides minus 1 to the perimeter (in this case each end square contributes 4-1--the 3 sides forming a cap

Our Conclusion

- Our claim that $p=(n-2)(s-2) + 2(s-1)$ is supported by our evidence to show that for any number of polygons lined up in a row, our rule works.

- Any questions?