EECS 270

Exam #1

For all questions, show all work that leads to your answer.

Problem #	Possible Points	Points Earned
1	20	
2	13	
3	30	
4	12	
5	17	
6	8	
Total	100	

1. (19 Points Total)

(a: 5 pts) Prove the following theorem of Boolean algebra using existing theorems and axioms. Do not use K-maps or perfect induction. Indicate whenever you use one of the following theorems: covering, consensus, combining.

$$\text{Theorem: } X + X'Y = X + Y$$

(b: 6 pts) Using the theorem you have just proven, reduce the following expression to two product terms. Do not use K-maps. Indicate whenever you use one of the following theorems: covering, consensus, combining.

$$ABC + C'D'E' + ABD + ABE$$

(c: 6 pts) Given the following function:

$$F = \sum_{X,Y,Z}(i_1, i_2, i_3) \; ; \;\; 0 \le i_1, i_2, i_3 \le 7 \; ; \; i_1 \ne i_2, i_2 \ne i_3, i_1 \ne i_3$$

What is the dual of F? *(Hint: pick three numbers for i1, i2, and i3 and work through the problem.)*

(d: 3pts) Given the function:

$F = \sum_{X,Y,Z} (i_1, \ldots, i_n)$; $0 \le n \le 8$, where $i_1, \ldots, i_n$ are not equal to each other.

For F to be self-dual, what must be the value of *n*? Explain your answer.

2. (13 Points Total) Perform the following number problems:

(a: 3 pts) $BF.AC12_{16} = ?_8$

(b: 3 pts) $233.61_{10} = ?_2$ Use 6 digits of precision to the right of the binary point.

(c: 3 pts) Write the following number using signed-magnitude, one's complement, and two's complement representation using 6-bit strings.

$-14_{10} = ?_2$

Signed Magnitude:

One's Complement:

Two's Complement:

(d: 4 pts) Perform the following two's complement subtraction using 6-bit strings in the same way that a computer would. Briefly explain why an overflow did or did not occur.

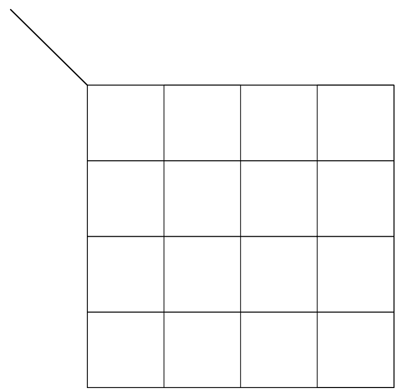
(14) subtract (-20)

3. (30 Points Total)

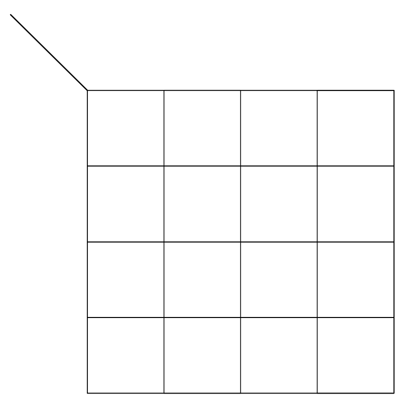
(a: 6 pts) Given the function:

$$F = \sum_{W,X,Y,Z}(0, 1, 4, 5, 7, 8, 12, 14, 15)$$

Represent F on the K-map below and identify all prime implicants. Mark all essential prime implicants with an “**X”**.



Redraw the K-map below after the essential prime implicants have been removed. Of the remaining prime implicants, mark any that can be eclipsed with a “●”.

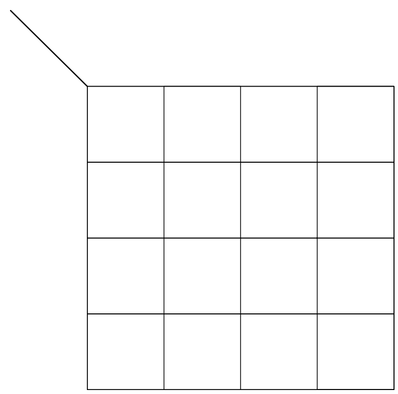


Derive the minimal SOP for F:

(b: 4 pts) Does your minimal SOP implementation contain any potential hazards? If so, identify the input transition(s) that could potentially cause a hazard.

W	X	Y	Z

(c: 6 pts) Derive the minimal POS for F.



Minimal POS:

Which is smaller in terms of number of literals, the SOP or the POS?

(d: 4 pts) You are now allowed to change any cell (whether it is currently 0 or 1) to a don't care. What is the optimal cell to place the don't care such that you obtain an expression for F (SOP or POS) with the least possible number of literals?

(e: 3 pts) Based on where you placed the don't care cell in part 3d, are the minimal SOP and POS for this new function equal? Briefly explain your answer.

(f: 7 pts) Find a function in four variables,

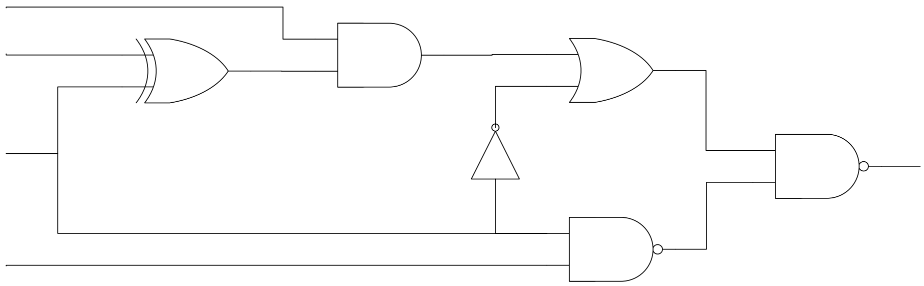
$$F = \sum_{W,X,Y,Z}(?, ?, \ldots)$$

such that the following conditions on F are met:

- The minimal SOP for F is equal to the canonical sum for F.
- The minimal POS for F is equal to the canonical product for F.

Given a function in *n* variables, how many functions that meet the above conditions exist?

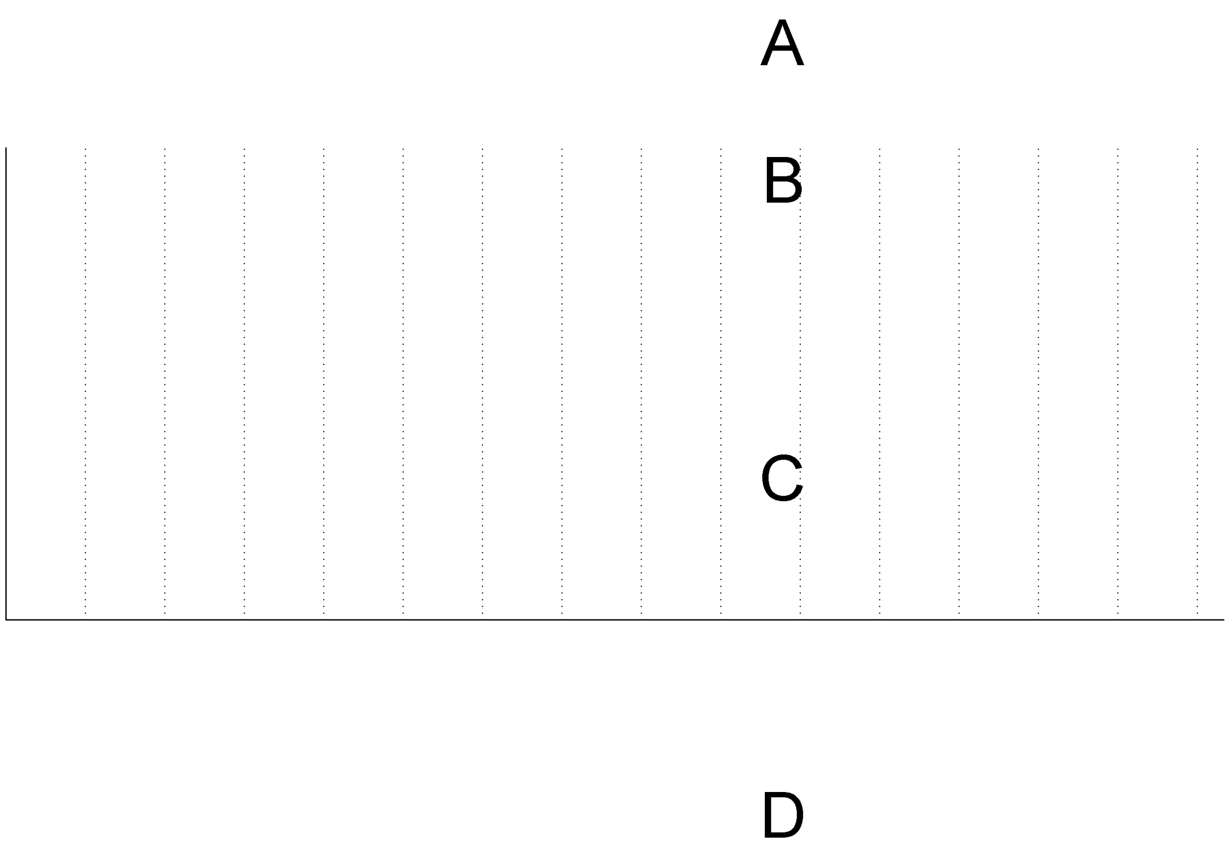
4. (12 Total Points) Consider the circuit below where each gate has a rising and falling delay of 1ns:



(a: 6 pts) Compute the delay $t_{pHL}^{C \to Q}$ and $t_{pLH}^{C \to Q}$ under the following input combination:

A = 0, B = 1, D = 1

Show your work on the timing diagram below, including all causality arrows.



(b: 6 pts) Is this circuit hazard free? If yes, explain why it is hazard free. If no, indicate which input must be switched and in which direction (rising/falling) and what the fixed input state of the other inputs must be.

5. (17 Points Total) Given the three valid codes from a 15-bit error detection/correction code:

010100000101001
001011001000010
100000110010100

(a: 3 pts) What is the hamming distance of this code?

(b: 4 pts) If you are interested only in detecting errors in a particular channel, what is the maximum number of errors that can exist in the channel, when using this code?

If you are interested in correcting all errors that could occur in a particular channel, what is the maximum number of errors that can exist in the channel?

(c: 5 pts) Consider a very unusual channel in which either exactly 1 bit is flipped or exactly *n* bits are flipped (where $n \geq 3$). What is the minimum required hamming distance to both detect and correct the errors in this channel?

(d: 5 pts) If *n* = 3 for the above channel, what is the minimum hamming distance necessary if we only want to detect errors?

6. (8 Points Total) Implement the following function using only inverters and two input, single select muxes. For full credit, use as few muxes as possible. Draw your diagram very clearly.

$$F = \sum_{X,Y,Z}(0, 2, 7)$$