# CFR and SNR Estimation Based on Complementary Golay Sequences for Single-Carrier Block Transmission in 60-GHz WPAN

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***Abstract*— The single-carrier block transmission (SCBT), *a.k.a.* single-carrier frequency-domain equalization (SC-FDE), is being considered as an option technique for the wireless personal area network (WPAN) operating at 60-GHz. To combat the inter-symbol interference (ISI) in non-line-of-sight (NLOS) environments, the frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion is much more effective than the one based on the zero forcing (ZF) criterion. However, MMSE-FDE needs the estimation of both the channel frequency response (CFR) and the signal-to-noise ratio (SNR). In this work, we estimated CFR and SNR based on the complementary Golay sequences whose sum auto-correlation has a unique peak and zero sidelobe. Through simulations, we proved that the proposed CFR and SNR estimation methods are very effective for SCBT system using MMSE-FDE.**

***Index Terms*— SCBT, SC-FDE, 60-GHz, WPAN, MMSE-FDE, ZF-FDE, complementary Golay sequences.**

## I. INTRODUCTION

There are two competitive transmission techniques for the millimeter-wave wireless personal area network (WPAN) operating at 60-GHz. One is the orthogonal frequency division multiplexing (OFDM) and the other is the single-carrier block transmission (SCBT), *a.k.a.* single-carrier frequency-domain equalization (SC-FDE) [1]-[3]. Compared with OFDM, SCBT has many advantages such as lower peak-to-average power ratio (PAPR) and less sensitivity to carrier frequency offset (CFO). Thus, SCBT is being considered as an option technique to support the multi-gigabit wireless data transmission in 60-GHz WPAN [4]-[10].

Usually, SCBT system depends on the frequency-domain equalization (FDE) [1]-[3] to combat the inter-symbol interference (ISI) caused by the multi-path fading channel. FDE can take the form of either zero forcing (ZF) or minimum mean square error (MMSE). MMSE-FDE has much better performance than ZF-FDE over multi-path fading channels [1]-[3][8][10]. However, MMSE-FDE has to know the signal-to-noise ratio (SNR) in addition to the channel frequency response (CFR).

In this work, we use the complementary Golay sequences [11]-[14] to estimate CFR and SNR. The Golay sequences are pair sequences with a prominent feature that the sum auto-correlation has a unique peak and zero sidelobe. This can effectively remove inter-symbol interference (ISI) to improve the accuracy of channel estimation.

We followed the *Selection Criteria* [15] to conduct the computer simulations. We proved that the CFR and SNR estimation methods based on the complementary Golay sequences are very effective for SCBT system using MMSE-FDE.

The remainder of this paper is organized as follows. In Section II, we introduce the SCBT system using MMSE-FDE. In Section III, the CFR and SNR estimation methods are presented respectively. The simulation results are shown in Section IV. Finally, we conclude this paper in Section V.

*Notation*: The vector and matrix variables are denoted by boldface letters. The time-domain and frequency-domain signals are denoted by lowercase and uppercase letters, respectively.

## II. SCBT USING MMSE-FDE

In Fig.1, we show the block diagram of SCBT system using MMSE-FDE [8][10].

Take the $n$-th block of the received signal as example. After guard interval (GI) removal, this $K$-length block (time domain) can be written in vector form

$$\mathbf{y}_n \triangleq (y[n,0] \cdots y[n,l] \cdots y[n,K-1]), \tag{1}$$

where $y[n,l]$ is the $l$-th symbol in the $n$-th block. For simplicity, we drop the block index $n$ and (1) becomes

$$\mathbf{y} \triangleq (y[0] \cdots y[l] \cdots y[K-1]). \tag{2}$$

Through $K$-point FFT, the time-domain vector $\mathbf{y}$ is transformed into a frequency-domain one

$$\mathbf{Y} \triangleq (Y[0] \cdots Y[l] \cdots Y[K-1]), \tag{3}$$

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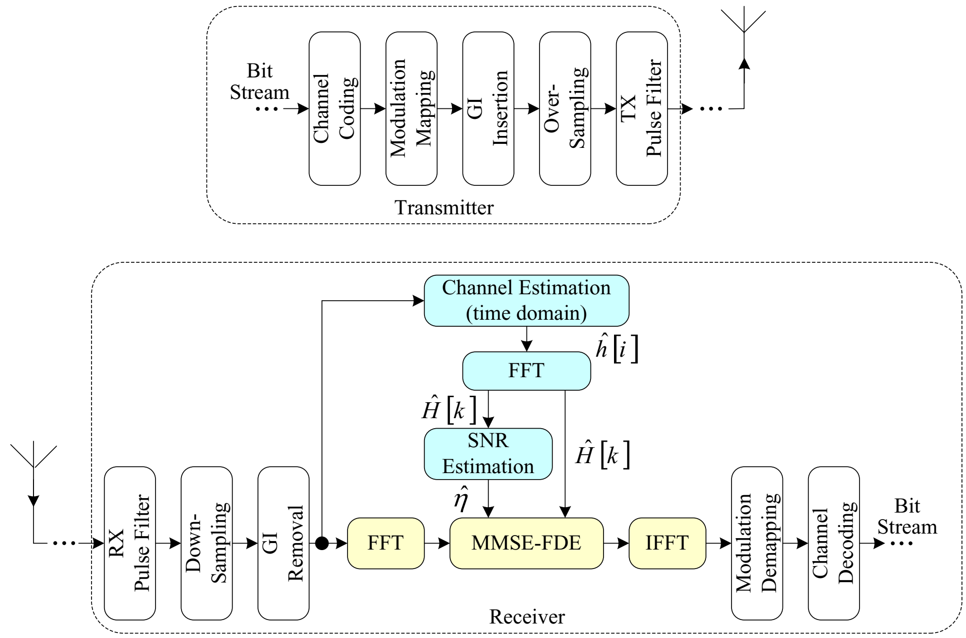


Fig. 1. Block diagram of SCBT system using MMSE-FDE.

where $Y[k]$ is the signal at the $k$-th subcarrier and can be written as

$$Y[k] = \sum_{l=0}^{K-1} y[l] F_K^{kl}, \tag{4}$$

$$F_K \triangleq e^{-j(2\pi/K)}.$$

Suppose the originally sent data block (time domain) is

$$\mathbf{x} \triangleq (x[0] \cdots x[l] \cdots x[K-1]), \tag{5}$$

and its corresponding frequency-domain signal vector is

$$\mathbf{X} \triangleq (X[0] \cdots X[l] \cdots X[K-1]). \tag{6}$$

Hence, the original and received frequency-domain signals (at the $k$-th subcarrier) has the relation

$$Y[k] = H[k]X[k] + W[k], \tag{7}$$

where $H[k]$ and $W[k]$ are the channel frequency response (CFR) and the noise at the $k$-th subcarrier, respectively.

FDE can be realized as a $K$-branch linear feed-forward equalizer with $C[k]$ as the complex coefficient at the $k$-th branch (subcarrier). The linear FDE can take the form of either zero forcing (ZF) or minimum mean-square error (MMSE). If optimized based on ZF criterion, the FDE coefficient $C[k]$ is

$$C_{zf}[k] = \frac{1}{H[k]}. \tag{8}$$

If optimized based on MMSE criterion, the FDE coefficient $C[k]$ becomes

$$C_{mmse}[k] = \frac{H[k]^*}{|H[k]|^2 + 1/\eta}, \tag{9}$$

where $\eta$, $*$, and $|\cdot|$ denote SNR, conjugate transpose and module of a complex value, respectively.

In severe frequency-selective fading where the spectral null (deep fading) occurs, the inversion of $H[k]$ in ZF-FDE may lead to infinity and results in noise enhancement at those frequencies of spectral null (deep fading). MMSE-FDE is more appealing since it can make compromise between the residual inter-symbol interference (ISI) (in the form of gain and phase mismatches) and noise enhancement. Therefore, it can minimize the combined effect of ISI and noise. This is particularly attractive for equalizing the channels of severe frequency-selective fading. MMSE-FDE has significantly better performance than ZF-FDE over multi-path fading channels [1]-[3][8][10].

However, as indicated by the equation (9), MMSE-FDE needs to estimate both CFR ($H[k]$) and SNR ($\eta$). We will introduce the estimation methods in Section III.

## III. CFR and SNR Estimation based on Complementary Golay Sequence

In this section, first we introduce the channel estimation sequence (CES) based on the complementary Golay sequence, then we present the CFR and SNR estimation methods.

### A. *Channel Estimation Sequence (CES) based on Complementary Golay Sequence*

The channel estimation sequence (CES) is based on complementary Golay sequences [11]-[14] which are made up of "a" and "b" parts. Each part has the length of $L = 2^B$ where $B$ is a positive integer. The most prominent feature is that the sum auto-correlation of the pair ("a" and "b") has a unique peak and zero sidelobe as shown in Fig.2. This can effectively remove the inter-symbol interference (ISI) to improve the accuracy of channel estimation. The auto-correlation can be implemented in a very simple structure of $B$ delay elements, $B$ inverters and $2B$ adders.

The CES based on complementary Golay sequences is shown in Fig.3. It has two parts and each part is made up of a cyclic prefix, a Golay sequence ("a" or "b") and a cyclic postfix. Both the cyclic prefix and cyclic postfix has the length of $M$.

The preamble used for channel estimation is made up of $G$ repetitions of CES.

### B. CFR Estimation

The CFR estimation method was presented in [8]. We briefly review this method in the following.

Suppose $K_0$ is the number of nonzero channel impulse response (CIR) taps, which depends on the delay profile of the wireless channels. The CIR vector estimated by the auto-correlation of "a" sequence is

$$\hat{\mathbf{h}}_{a,g} \triangleq \left(\hat{h}_a[g,0],\cdots,\hat{h}_a[g,i],\cdots,\hat{h}_a[g,K_0-1]\right), \quad (10)$$

where $\hat{h}_a[g,i]$ is the $i$-th CIR tap estimated by "a" sequence in the $g$-th CES. Similarly, the CIR vector estimated by "b" sequence is

$$\hat{\mathbf{h}}_{b,g} \triangleq \left(\hat{h}_b[g,0],\cdots,\hat{h}_b[g,i],\cdots,\hat{h}_b[g,K_0-1]\right). \quad (11)$$

Due to the sum auto-correlation property of complementary Golay sequence, we average $\hat{\mathbf{h}}_{a,g}$ and $\hat{\mathbf{h}}_{b,g}$ to get the CIR vector (estimated by the $g$-th CES), i.e.,

$$\begin{aligned}\hat{\mathbf{h}}_g &\triangleq (\hat{\mathbf{h}}_{a,g}+\hat{\mathbf{h}}_{b,g})/2 \quad (12)\\ &\triangleq \left(\hat{h}[g,0],\cdots,\hat{h}[g,i],\cdots,\hat{h}[g,K_0-1]\right).\end{aligned}$$

The channel frequency response (CFR) at the $k$-th subcarrier can be generated by FFT

$$\hat{H}[g,k]=\sum_{i=0}^{K_0-1}\hat{h}[g,i]F_K^{ki}. \quad (13)$$

Averaging the $G$ estimates at the $k$-th subcarrier, we have the averaged CFR

$$\hat{H}[k]=\frac{1}{G}\sum_{g=1}^{G}\hat{H}[g,k]. \quad (14)$$

### C. SNR Estimation

We proposed a simple SNR estimation method based on the estimated CFR in [10]. The noise at the $k$-th subcarrier in the $g$-th CES is

$$\hat{W}[g,k]=\hat{H}[g,k]-\hat{H}[k]. \quad (15)$$

The signal power can be estimated as

$$P_s=\frac{1}{K}\sum_{k=1}^{K}|\hat{H}[k]|^2, \quad (16)$$

and the noise power can be estimated as

$$P_n=\frac{1}{GK}\sum_{g=1}^{G}\sum_{k=1}^{K}|\hat{W}[g,k]|^2. \quad (17)$$

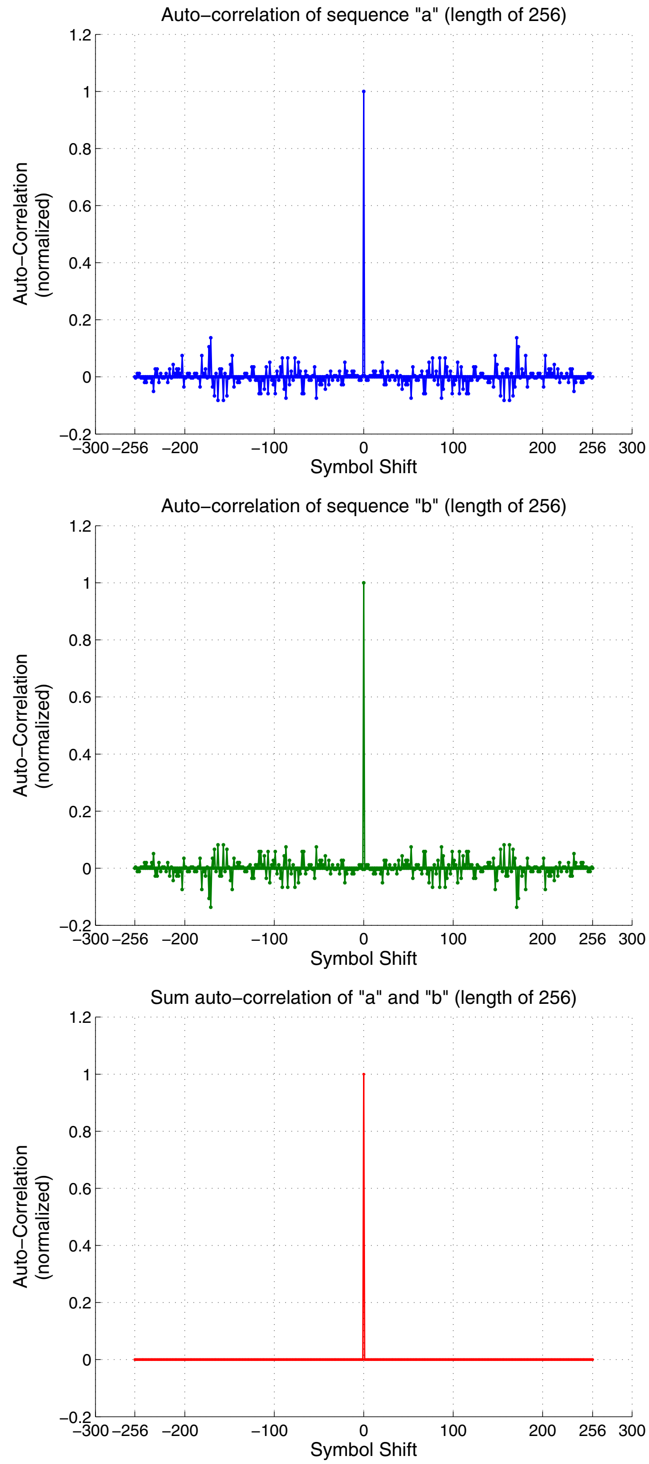


Fig. 2. Auto-correlations of sequence "a" and "b", and their sum auto-correlation.

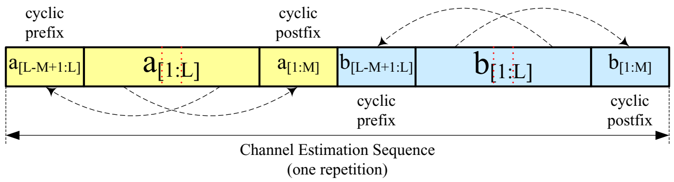


Fig. 3. Channel estimation sequence (CES) based on complementary Golay sequences.

Thus, we have the estimated SNR

$$\hat{\eta} = \frac{P_s}{P_n}. \tag{18}$$

Substituting $\hat{H}[k]$ and $\hat{\eta}$ into (9), we obtain the MMSE-FDE coefficient

$$\hat{C}_{mmse}[k] = \frac{\hat{H}[k]^*}{|\hat{H}[k]|^2 + 1/\hat{\eta}}. \tag{19}$$

Transforming the equalized frequency-domain signal back into the time domain by IFFT, we have

$$\tilde{y}[k] = \sum_{l=0}^{K-1} \hat{C}_{mmse}[l] Y[l] F_K^{-kl}. \tag{20}$$

Then, the equalized time-domain signal block

$$\tilde{\mathbf{y}} \triangleq (\tilde{y}[0] \cdots \tilde{y}[k] \cdots \tilde{y}[K])$$

is sent forward for demodulation and decoding.

## IV. Simulation Results

We investigated the effects of CFR and SNR estimation on the performance of LDPC coded SCBT system in 60-GHz WPAN environments.

The system and simulation parameters are shown in Table I. The multi-path fading channel used in the simulation was the *NLOS Residential* model which is the mandatory channel model required by the *Section Criteria* [15]. The symbol rate (Nyquist bandwith), roll-off factor and channel bandwidth came from the channelization plan [15]. We used QPSK as modulation and the irregular LDPC (576,432) of coding rate ($R$) 3/4 as channel coding (the irregular LDPC codes are designed by using the structured LDPC design method [17]). The maximum decoding iteration number was set to 64. The FFT / IFFT length was 256. The guard interval (GI), which was unique word [16], had length of 64. The Golay sequence length ($L$) was 256. The length ($M$) of the cyclic prefix and cyclic postfix in CES was 128. The synchronization was assumed to be ideal. The FDE was based on MMSE algorithm [10]. We also simulated ZF-FDE for comparison. Each data packet was loaded with 2K (2048) raw bytes. The other parameters not shown in Table I followed the requirements in the *Section Criteria* [15].

As indicated by the equations (8) and (9), ZF-FDE only depends on CFR ($H[k]$) while MMSE-FDE depends on both CFR ($H[k]$) and SNR ($\eta$).

To investigate their independent effects on the performance, both CFR and SNR estimations were assumed to have two states: ideal and estimated. In Table II, we show the abbreviation definitions of the CFR and SNR estimation states, which were used in the following BER figures.

TABLE I
System and Simulation Parameters

| | |
|---|---|
| Channel model | NLOS Residential [15] (RMS delay=6.24ns) |
| Symbol rate (Nyquist bandwith) | 1.632GHz |
| Roll-off factor | 0.3235 |
| Channel bandwidth | 2.16GHz |
| Modulation | QPSK |
| Channel coding | Irregular structured LDPC codes (576,432), $R$=3/4 |
| Max. decoding iterations | 64 |
| FFT / IFFT length (time) | 256 (156.86ns) |
| GI (UW) length (time) | 64 (39.22ns) |
| Golay sequence length ($L$) | 256 |
| Cyclic prefix / postfix length ($M$) | 128 |
| Synchronization | Ideal |
| CFR estimation | Ideal / estimated |
| SNR estimation | Ideal / estimated |
| FDE | MMSE-FDE (ZF-FDE for comparison) |
| Packet size | 2048 bytes (16384 bits) |

TABLE II
Abbreviation Definitions of CFR and SNR Estimation States

| | |
|---|---|
| CFR-ide | Ideal CFR |
| CFR-est | Estimated CFR |
| SNR-ide | Ideal SNR |
| SNR-est | Estimated SNR |

In Figs.4-6, we show the BER performance of LDPC coded SCBT system in three scenarios:

- "CFR-ide" and "SNR-est" (Fig.4);
- "CFR-est" and "SNR-ide" (Fig.5);
- "CFR-est" and "SNR-est" (Fig.6).

From the BER figures, we have the following observations.

- *Effects of SNR estimation*
  In Fig.4, we observed that the proposed SNR estimation method has very minor effects on MMSE-FDE performance. Even as we use very small CES repetition number, e.g., $G$=2, the performance degradation can almost be neglected.
- *Effects of CFR estimation*
  In Figs.5 and 6, we show the performance when CFR is estimated. We observed that the performance degradation of MMSE-FDE is mainly caused by CFR estimation. For example, as CES repetition number G is 2, the performance degradation is about 0.7dB (at BER of $10^{-6}$). However, this degradation can be significantly reduced by increasing CES repetition number $G$. For example, as $G$ is increased to 4, the performance degradation can be almost neglected.

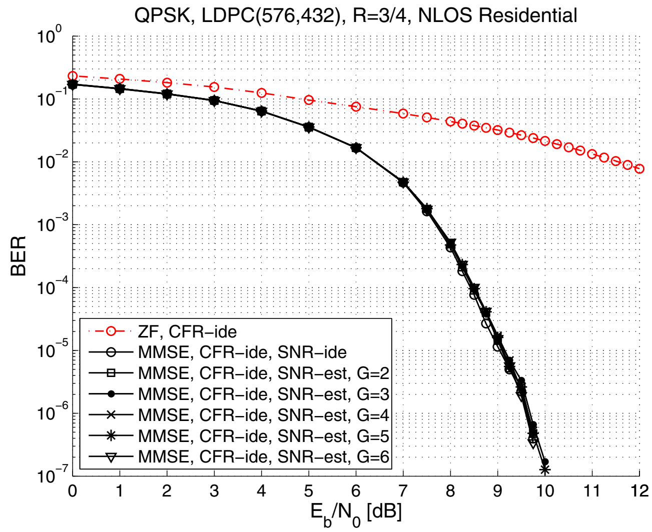


Fig. 4. BER of LDPC coded SCBT system using ideal CFR ("CFR-ide") and estimated SNR ("SNR-est").

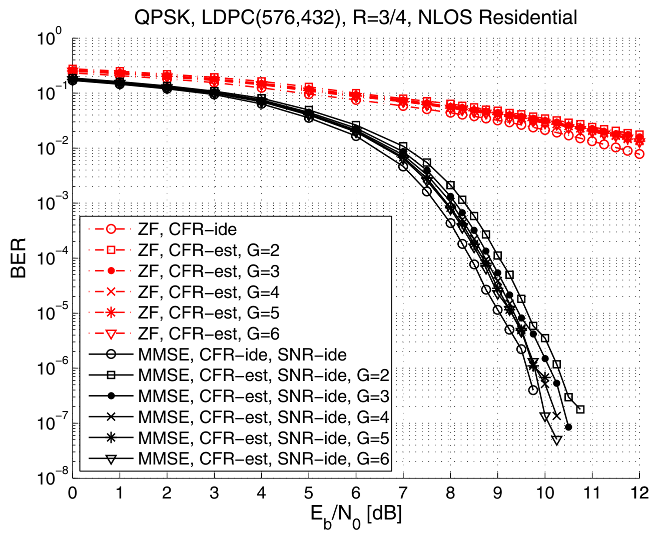


Fig. 5. BER of LDPC coded SCBT system using estimated CFR ("CFR-est") and ideal SNR ("SNR-ide").

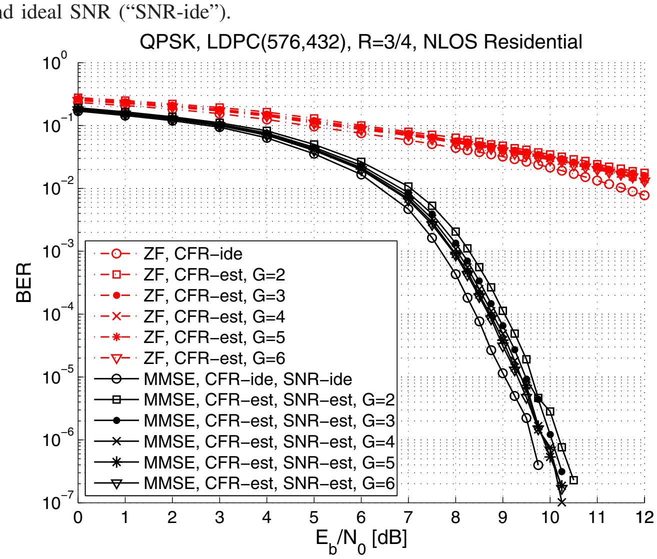


Fig. 6. BER of LDPC coded SCBT system using estimated CFR ("CFR-est") and estimated SNR ("SNR-est").

We also observed that increasing $G$ can not make ZF-FDE more robust against multi-path fading. MMSE-FDE is the most effective equalization method for SCBT system over multi-path fading channels.

## V. Conclusions

We proposed using the complementary Golay sequences to estimate CFR and SNR for LDPC coded SCBT system. The prominent feature of these sequences is that the sum auto-correlation has a unique peak and zero sidelobe. This can effectively remove ISI to improve the channel estimation accuracy. By conducting simulations in 60-GHz WPAN environment, we proved that the proposed methods are very effective for SCBT system.

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